

## AS **Mathematics**

MPC1 – Pure Core 1 Mark scheme

6360 June 2016

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

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Further copies of this mark scheme are available from aga.org.uk

## Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
Α	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and
	accuracy
Е	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

## **No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$y = \pm \frac{5}{3}x + \dots$	M1		
	$m=-\frac{5}{3}$	<b>A1</b>	2	$y = -\frac{5}{3}x - 1$ for guidance <u>must</u> see $m = \dots$ or statement such as "AB has gradient $-\frac{5}{3}$ so line parallel to AB
				also has gradient $-\frac{5}{3}$ ."
(b)	5x+3y+3=0 & 3x-2y+17=0  eg 10x+6+9x+51=0	M1		correct equations used and correct elimination of x or y eg $19x+57=0$ or $19y-76=0$ etc
	$x = -3$ or $x = -\frac{57}{19}$ or $y = 4$ or $y = \frac{76}{19}$	<b>A1</b>		either x or y correct in any equivalent form
	{both $x = -3$ and $y = 4$ } or $(-3, 4)$	<b>A1</b>	3	both coordinates written as integers
(c)	5(2k+3)+3(4-3k)+3=0 10k+15 + 12-9k +3=0	M1		correct substitution into correct equation & correct expansion of brackets
	k = -30	A1	2	
	Total		7	

(a) Do not penalise incorrect rearrangement if  $m = -\frac{5}{3}$  is stated.

**Example**  $y = -\frac{5}{3}x - 3$  so  $m = -\frac{5}{3}$  scores **M1 A1** 

**NMS**  $m = -\frac{5}{3}$  earns 2 marks. **NMS**  $-\frac{5}{3}$  earns **M1 A0** . **NMS**  $(m =) \frac{5}{3}$  earns **M1 A0** .

**NMS** Award **M1 A0** only for " $m = -\frac{5}{3}x$ ".

**(b)**  $5\left(\frac{2y}{3} - \frac{17}{3}\right) + 3y + 3 = 0$  earns **M1**, however  $5\left(\frac{2y}{3} + \frac{17}{3}\right) + 3y + 3 = 0$ , for example, scores **M0**.

Other examples scoring **M1** are  $3x - 2\left(-\frac{5}{3}x - 1\right) + 17 = 0$ ;  $-\frac{5}{3}x - 1 = \frac{3}{2}x + \frac{17}{2}$ 

Accept any correct equivalent fraction for first A1 but must have both x = -3 and y = 4 for final A1.

NMS (-3,4) scores 3 marks

Q2	Solution	Mark	Total	Comment
(a)	45	B1	1	
(b)	$\frac{**+\sqrt{5}}{7+3\sqrt{5}} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}}$	M1		
	(Numerator = ) $315 + 7\sqrt{5} - 135\sqrt{5} - 15$	<b>A1</b>		at least this far
	(Denominator = $49 + 21\sqrt{5} - 21\sqrt{5} - 45$ ) = 4	B1		must be seen as denominator
	Value = $\frac{300 - 128\sqrt{5}}{4}$ = $75 - 32\sqrt{5}$	A1cso	4	
	Total		5	

## NO MISREADS ALLOWED IN THIS QUESTION

Condone multiplication by  $7-3\sqrt{5}$  instead of  $\times \frac{7-3\sqrt{5}}{7-3\sqrt{5}}$  for **M1 only if** subsequent working shows multiplication by both numerator and denominator – otherwise **M0** 

For first A1  $45 \times 7$ ,  $45 \times 3$  and  $3 \times 5$  must be evaluated correctly

An error in the denominator such as  $49+7\sqrt{5}-7\sqrt{5}-45=4$  should be given **B0** and it would then automatically lose the final **A1cso** 

May use alternative conjugate  $\times \frac{3\sqrt{5}-7}{3\sqrt{5}-7}$  M1; numerator =  $-315-7\sqrt{5}+135\sqrt{5}+15$  A1 etc

		Mark	Total	Comment
(a)(i) $\left(x - \frac{7}{2}\right)$		M1		$(x-3.5)^2$ <b>OE</b>
	$\left(x-\frac{7}{2}\right)^2-\frac{41}{4}$	<b>A1</b>	2	$(x-3.5)^2-10.25$
(ii) (Minii	mum value =) −10.25 <b>OE</b>	B1F	1	<b>must FT</b> their $q$
(b) Transl	ation	<b>E</b> 1		or translate(d) (by/through) (and no other transformation given)
[0.5]		M1		(and no other transformation given)
	$\begin{bmatrix} 0.5\\10.25 \end{bmatrix}$	<b>A1</b>	3	must express as vector to earn A1 mark
	Total		6	

- (a)(i) If M1 is not earned, award SC1 for  $\left(x \frac{7}{2}\right) \frac{41}{4}$ 
  - (ii) Do **NOT** accept any **pair** of values. **Example** (3.5, -10.25) scores **B0** since this is hedging bets Condone y = "their" q for **B1** but x = "their" q scores **B0**
  - (b) Do NOT accept "shift", "move", "slide", "transformation", "trans" etc for E1 Accept "0.5 in x-direction", " $\frac{1}{2}$  to the right", "(0.5,\*)" for M1 only

Q4	Solution	Mark	Total	Comment
(a)(i)	$(p(-3)) = (-3)^3 - 5(-3)^2 - 8(-3) + 48$	M1		clear attempt at p(-3) NOT long division
	= -27 - 45 + 24 + 48 = 0			must see powers of $-3$ simplified correctly working showing that $p(-3)=0$
	therefore $x+3$ is a factor	A1	2	and correct statement
(ii)	$x^2 + bx + c$ with $b = -8$ or $c = 16$	M1		by inspection
	$x^2 - 8x + 16$	<b>A1</b>		may see as quotient in long division
	(p(x) =) (x+3)(x-4)(x-4)	<b>A1</b>	3	must see product
(b)(i)	$p(2) = 2^3 - 5 \times 2^2 - 8 \times 2 + 48$ $= 8 - 20 - 16 + 48$	M1		clear attempt at p(2) NOT long division
	(Remainder =) 20	<b>A1</b>	2	
(ii)	Quadratic factor $x^2 + bx + c$			
	b = -3 or $c = -14$	<b>M1</b>		by inspection
	$x^2 - 3x - 14$	<b>A1</b>		may see as quotient in long division
	$(p(x) =) (x-2)(x^2-3x-14)+20$	<b>A1</b>	3	must see full correct expression
	Total		10	

(a)(i) Minimum required for statement is "∴ factor"

Powers of -3 must be evaluated: **Example** "p(-3) = -27 - 45 + 24 + 48 = 0 so factor" scores **M1 A1** Statement may appear first :

**Example** "x+3 is factor if p(-3) = 0 & p(-3) = -27 - 45 + 24 + 48 = 0" scores M1 A1

However, **Example** "p(-3) =  $(-3)^3 - 5(-3)^2 - 8(-3) + 48 = 0$  therefore x+3 is a factor" scores **M1 A0** 

(ii) M1 may also be earned for a full long division attempt by (x+3), or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .

M1 may also be earned for *showing* p(4) = 0 and *stating* that (x-4) is a factor

**NMS**  $p(x) = (x+3)(x-4)^2$  scores **3 marks**;

- (b)(i) Do not apply ISW for eg "p(2) = 20, therefore remainder is -20" May use "their" product of factors p(2) = (2+3)(2-4)(2-4) for **M1** and **A1** if factors and working are all correct giving 20.
  - (ii) M1 may be earned for a full long division attempt by (x-2), or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .

M1 may also be earned for using their value from part (b)(i) for r and a full attempt to find b and c.

Q5	Solution	Mark	Total	Comment
(a)	$(x-5)^2 + (y+3)^2 = \dots$	M1		or $(x-5)^2 + (y-3)^2 =$
(ω)	$7^2 + 4^2$ or $49 + 16$ or $65$	B1		or seen under square root
		A1	3	<u> </u>
	$(x-5)^2 + (y+3)^2 = 65$	AI	3	or $(x-5)^2 + (y-3)^2 = 65$
(b)	$x_R = 12$	B1		
	$y_{R} = -7$	B1	2	B(12,-7)
	$y_B = -i$	DI	_	
	13	3.54		condone one sign error in one term
(c)	Grad $AC = \frac{13}{-2 - 5}$	M1		FT their B if grad AB or grad BC is used.
	$= -\frac{4}{7}$	<b>A1</b>		
	Grad tgt = $\frac{7}{4}$	B1F		
	4	DII		
	Equation of tgt: $y-1 = "their" \frac{7}{4}(x-2)$	m1		or $y = "their" \frac{7}{4}x + c$ & attempt to find c
	4 4			<b>T</b>
				using $x = -2$ and $y = 1$
	7x - 4y + 18 = 0	<b>A1</b>	5	any multiple – must have integer
				coefficients and all terms on one side
(d)	$CT^2 = AT^2 + AC^2$			
(4)	$(CT^2 -)$ $A^2 + "their" 65$	M1		Pythagoras with hyp=CT
	(CI -) 4 + men 03	1,11		& $AC^2$ = "their" k or correct
	$(CT^{2} =) 4^{2} + "their" 65$ $(CT^{2} =) 81$	<b>A1</b>		or $(CT =)\sqrt{81}$
	(CT = )9	<b>A1</b>	3	all notation correct; must simplify $\sqrt{81}$
				an notation correct, must simplify voi
	Total		13	
	10141			

(a) NMS  $(x-5)^2 + (y+3)^2 = 65$  scores 3 marks

allow RHS =  $\left(\sqrt{65}\right)^2$  instead of 65 for full marks

**Example:**  $(x-5)^2 + (y+3)^2 = \sqrt{65}$  earns **M1 B1 A0** 

Equation of circle must be written explicitly as  $(x-5)^2 + (y+3)^2 = 65$  to earn A1 mark

(c) Award M1 A0 for grad AC = 4/7

For **m1** candidate must be attempting equation of tangent; if **B1F** not earned and their gradient of AC is m then award **m1** if using 1/m or -m and correct coordinates (-2,1).

For final A1 accept answers such as 0 = 8y - 14x - 36 but NOT 7x - 4y = -18

(d) Example:  $4^2 + 65 = 81 = 9$  scores M1, A1, A0; Example:  $4^2 + 65 = 81$ ,  $\sqrt{81} = 9$  scores M1, A1, A1

Q6	Solution	Mark	Total	Comment
(a)(i)	$(x=)$ $\frac{4 \pm \sqrt{80}}{-4}$ , or $(x=)$ $\frac{-4 \pm \sqrt{80}}{4}$ or $(x=)$ $\frac{-2 \pm \sqrt{20}}{2}$	M1		if completing square must have at least $x+1=\pm\sqrt{5}$
	$(x=) -1 \pm \sqrt{5}$	<b>A1</b>	2	do <b>not</b> accept $-1 \pm -\sqrt{5}$ for <b>A1</b>
(ii)	8	M1 A1	2	shape as shown in all 4 quadrants, max to left of y-axis with y-intercept 8 stated/marked
(b)(i)	$k(x+4) = 8-4x-2x^{2}$ $2x^{2} + kx + 4x + 4k - 8 = 0$ $2x^{2} + (k+4)x + 4(k-2) = 0$	B1	1	must expand $k(x+4)$ & have all terms on one side with =0 before final line <b>AG</b> be convinced
(ii)	$(k+4)^2 - 4 \times 2 \times 4(k-2)$ (=0) $k^2 - 24k + 80$ (=0) k = 4, $k = 20$	M1 A1 A1cso	3	correct discriminant
	Total		8	

(a)(ii) Withhold A1 if maximum y- value is clearly not greater than 8, or graph has wrong curvature in third and fourth quadrants.

Do not withhold **A1** if incorrect *x*-intercepts are marked on *x*-axis, etc.

Accept (0,8) stated or marked on y-axis as y-intercept, but do NOT accept (8,0).

**(b)(i)** Must have "=0" on final line but this may be on LHS.

Do not accept incorrect "trailing equal" signs, ie from line 1 to line 2 of proof.

(ii) Condone poor use/omission of brackets for M1 if correct discriminant is intended, but the A1 cso cannot then be earned even if recovered later. Candidates must have "= 0" on at least one line of working or statement " $b^2 - 4ac = 0$ " and all working correct to earn A1cso.

If candidate uses "> 0" etc then withhold **A1cso** even if final answer is written as k = 4, k = 20.

Q7	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right) -2x - 9x^2$	M1 A1		one term correct all correct ( no +c etc)
	when $x = -2$ , $\frac{dy}{dx} = (4 - 36 =) - 32$	<b>A1</b>		
	y = "their - 32" x + c & attempt to find $c$ using $x = -2$ and $y = 24$	m1		or $y-24 = "their - 32"(x2)$
	y = -32x - 40	<b>A1</b>	5	must write in this form; no <b>ISW</b> here
(ii)	$y = 0 \Rightarrow x = -\frac{5}{4} \text{ OE}$	B1F	1	strict FT from their answer to (a)(i)
(b)(i)	$4x - \frac{x^3}{3} - \frac{3x^4}{4} (+c)$	M1 A1		two terms correct all correct
	$\left[4 \times 1 - \frac{1^{3}}{3} - \frac{3 \times 1^{4}}{4}\right] - \left[4 \times (-2) - \frac{(-2)^{3}}{3} - \frac{3(-2)^{4}}{4}\right]$	m1		"their" F(1) – F(–2)
	$\left[4 - \frac{1}{3} - \frac{3}{4}\right] - \left[-8 + \frac{8}{3} - \frac{48}{4}\right]$	<b>A1</b>		correct with powers of 1 and (-2) and minus signs handled correctly
	$=20\frac{1}{4}$	<b>A1</b>	5	$20.25 , \frac{81}{4}, \frac{243}{12}$ <b>OE</b>
(ii)	Area of missing triangle = $(\frac{1}{2} \times 24 \times \frac{3}{4})$ = 9	<b>B1</b>		or correct <b>single</b> equivalent fraction
	Area of region = "their" ( <b>b</b> )( <b>i</b> ) – "their" $\Delta$	M1		"their" $(20\frac{1}{4}-9)$
	$=11\frac{1}{4}$	<b>A1</b>	3	11.25, $\frac{45}{4}$ , $\frac{135}{12}$ <b>OE</b>
	Total		14	
-	10101			

(a)(i) Must see y = -32x - 40 explicitly for final A1; ie not enough to see y = -32x + c with c = -40 appearing on later line.

(a)(ii) Allow 
$$-\frac{40}{32}$$
 etc.

(b)(i) Must combine terms for final A1; Example ...  $3\frac{1}{4} + 17$  scores final A0.

(ii) May find triangle area by considering trapezium with one side of zero length or integration for B1.
 For M1 condone use of "their" Δ -"their" (b)(i) if appropriate for their values.
 Be generous in awarding this M1 provided you are convinced they are considering the area of a triangle.

Q8	Solution	Mark	Total	Comment
(a)(i)	$\left(\frac{d^2y}{dx^2}\right) = 27 - 12x$	M1 A1	2	one term correct all correct (no $+c$ etc)
(ii)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 54 + 27 \times \left(-\frac{3}{2}\right) - 6 \times \left(-\frac{3}{2}\right)^2$	M1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 54 - \frac{81}{2} - \frac{54}{4} = 0$	<b>A1</b>		convincingly showing $\frac{dy}{dx} = 0$ and $\frac{dy}{dx} =$
	$\left(\frac{d^2y}{dx^2}\right) = 27 - 12 \times \left(\frac{-3}{2}\right)$	M1		<b>must</b> appear on at least one line correct substitution into "their", $\frac{d^2y}{dx^2}$
	$\frac{d^2 y}{dx^2} = 27 + 18 \ (= 45) > 0$			correct working and $\frac{d^2y}{dx^2}$ used and value
	$\Rightarrow P$ is minimum point	A1cso	4	shown to be > 0 with correct statement(s)
				must earn 3 previous marks to earn A1cso
(b)(i)	(Decreasing so) $54 + 27x - 6x^2 < 0$ $6x^2 - 27x - 54 > 0$	M1		
	$2x^2 - 9x - 18 > 0$	<b>A1</b>	2	AG be convinced
(ii)	(2x+3)(x-6)	M1		correct factors or correct use of formula as far as $\frac{9 \pm \sqrt{225}}{4}$
	CVs are $x = -\frac{3}{2}, x = 6$	<b>A1</b>		condone equivalent fractions here
	$\frac{+}{-\frac{3}{2}}$ $\frac{+}{6}$	M1		use of sign diagram or graph
	$x < -\frac{3}{2},  x > 6$	<b>A1</b>	4	fractions must be simplified for final mark no <b>ISW</b> here
	Total		12	IIO 10 17 IICIC
	Total			

(b)(ii) For second M1, if critical values are correct then sign diagram or sketch must be correct with correct CVs marked.

However, if CVs are not correct then second M1 can be earned for attempt at sketch or sign diagram but their CVs MUST be marked on the diagram or sketch.

Final A1, inequality must have x and no other letter.

Final answer of  $x < -\frac{3}{2}$  OR x > 6 (with or without working) scores **4 marks**. (A)  $k < -\frac{3}{2}$ , k > 6 (B)  $x < -\frac{3}{2}$  AND x > 6 (C)  $x \le -\frac{3}{2}$ ,  $x \ge 6$ 

(A) 
$$k < -\frac{3}{2}$$
,  $k > 6$  (B)  $x < -\frac{3}{2}$  AND  $x > 6$  (C)  $x \le -\frac{3}{2}$ 

with or without working, each score 3 marks (SC3)

**Example NMS**  $x < \frac{3}{2}$ , x > 6 scores **M0** (since one CV is incorrect)

**Example NMS** x > -1.5, x > 6 scores **M1 A1 M0** (since both CVs are correct)